



Meshless Methods

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Coupled EFG/FEM:

- Proposed by Krongauz/Belytschko (1996) to impose essential boundary conditions.
- MLS shape functions do not provide interpolation, while FE does so.

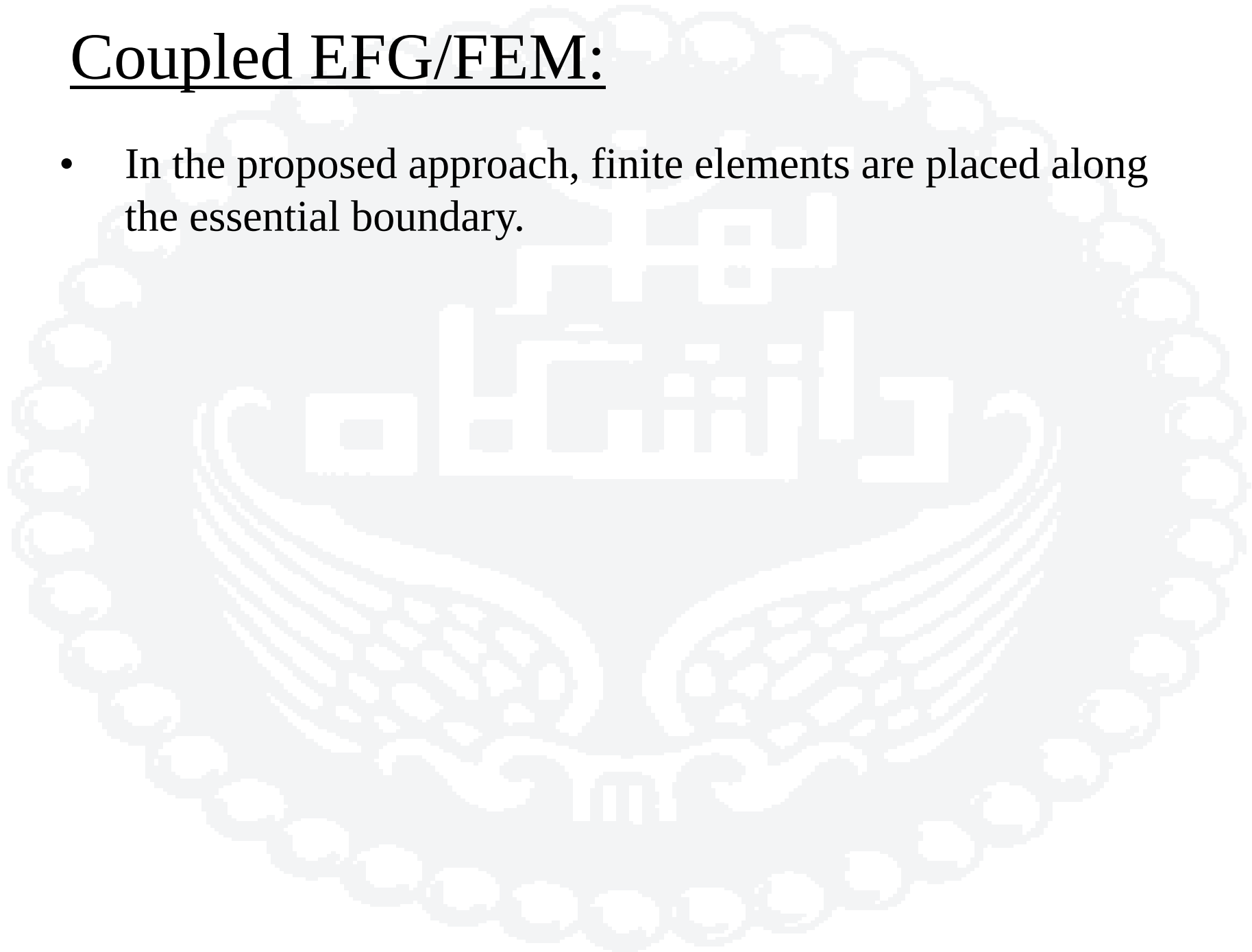
$$\phi_I(x_J) \neq \delta_{IJ}$$

$$u^h(x_J) = \sum_{I=1}^n \phi_I(x_J) u_I \neq u_J$$

- Moreover, along a boundary, the sum of all shape functions associated with nodes on the boundary is not equal 1.

Coupled EFG/FEM:

- In the proposed approach, finite elements are placed along the essential boundary.



Coupled EFG/FEM:

- For FE and EFG approximations:

$$u^{EFG}(x) = \sum_{I=1}^n \phi_I(x) u_I$$

$$u^{FE}(x) = \sum_{I=1}^4 N_I(x) u_I$$

ϕ_I MLS shape functions

N_I FE shape functions

Coupled EFG/FEM:

- The modified approximation can then be defined as:

$$\hat{u}^h(x) = \begin{cases} u^{EFG}(x) + v(x)[u^{FE}(x) - u^{EFG}(x)] & x \in \Omega_B \\ u^{EFG}(x) & x \in \Omega_E \end{cases}$$

$$v(x) = \sum_{I=1}^{n_b} N_I(x)$$

- n_b is the set of finite element nodes that are on the essential boundary, i.e. the sum is taken over the FE nodes that are on the essential boundary.
- $v(x)$ will be unity along the boundary and will vanish on the interface between boundary region Ω_B and EFG region Ω_E .

Coupled EFG/FEM:

- EFG/FE approximation is continuous, because at the side of any finite element adjacent to the EFG region: $v(x)=0$ and so:

$$\hat{u}^h(x) = u^{EFG}(x)$$

- Derivative of $\hat{u}^h(x)$ is discontinuous, and it undergoes a jump across the interface with magnitude of the jump for point x on the interface being equal to:

$$v_{,I}(x)[u^{FE}(x) - u^{EFG}(x)]$$

- The derivatives are also discontinuous across the element boundaries.

Coupled EFG/FEM:

- The new approximation can be written as:

$$\hat{u}^h(x) = \sum_{I=1}^n \hat{\phi}_I(x) u_I$$
$$\hat{\phi}_I(x) = \begin{cases} (1-v(x))\phi_I(x) + v(x)N_I(x) & x \in \Omega_B \\ \phi_I(x) & x \in \Omega_E \end{cases}$$

Coupled EFG/FEM:

- A linear field can be exactly reproduced

let $u_I = x_I$:

$$\hat{u}^h = \sum_{I=1}^n \hat{\phi}_I(x) x_I = \sum_{I=1}^n (1-v(x)) \phi_I(x) x_I + \sum_{I=1}^n v(x) N_I(x) x_I$$

$$\sum_{I=1}^n \phi_I(x) x_I^l = x_I^l \quad , l \leq m$$

$$\sum_{I=1}^n N_I(x) x_I = x_I$$

$$\hat{u}^h = (1-v(x)) x_I + v(x) x_I = x_I$$

Coupled EFG/FEM:

- This approximation represents a weighting scheme for EFG and FE interpolations.
- The closer the evaluation point is to the essential boundary, the closer the approximation becomes to finite elements.
 - On the boundary the approximation is equivalent to FE.
- The further away the evaluation point is from the essential boundary, the more the approximation becomes to EFG.
 - At the interface of boundary and EFG regions, the approximation is completely EFG.
- Similar procedure can be used for other meshless methods such as SPH.

Coupled EFG/FEM:

- The weak form can be written as (noting that the essential boundary conditions are automatically satisfied):

$$\int_{\Omega} \delta \varepsilon^T \sigma \, d\Omega - \int_{\Omega} \delta u^T b \, d\Omega - \int_{\Gamma_t} \delta u^T \bar{t} \, d\Gamma = 0$$

- Using the EFG/FE approximation :

$$u(x) = \sum_{I=1}^n \hat{\phi}_I(x) u_I$$
$$\delta u(x) = \sum_{I=1}^n \hat{\phi}_I(x) \delta u_I$$

- The discretized system is then obtained:

$$KU = F$$

Coupled EFG/FEM:

$$KU = F$$

$$K = \bigcap_{I,J=1}^{n_t} K_{IJ} \quad K_{IJ} = \int_{\Omega} B_I^T C B_J d\Omega$$

$$F = \bigcap_{I=1}^{n_t} f_I \quad f_I = \int_{\Omega} \hat{\Phi}_I^T b d\Omega + \int_{\Gamma_t} \hat{\Phi}_I^T \bar{t} d\Gamma$$

$$\hat{\Phi}_I = \begin{bmatrix} \hat{\phi}_I & 0 \\ 0 & \hat{\phi}_I \end{bmatrix} \quad B_I = \begin{bmatrix} \hat{\phi}_{I,x} & 0 \\ 0 & \hat{\phi}_{I,y} \\ \hat{\phi}_{I,y} & \hat{\phi}_{I,x} \end{bmatrix}$$

Coupled EFG/FEM:

$$\hat{\phi}_I(x) = \begin{cases} (1-v(x))\phi_I(x) + v(x)N_I(x) & x \in \Omega_B \\ \phi_I(x) & x \in \Omega_E \end{cases}$$

$$\hat{\phi}_{I,J}(x) = \begin{cases} (1-v(x))\phi_{I,J}(x) - v_{,J}(x)(\phi_I(x) - N_I(x)) + v(x)N_{I,J} & x \in \Omega_B \\ \phi_{I,J}(x) & x \in \Omega_E \end{cases}$$

